$$\int x^2 e^{3x}\,dx = \left| \begin{array}{ll} u = x^2 & du = 2x\,dx \\ dv = e^{3x}dx & v = \int e^{3x}dx = \frac{1}{3}e^{3x} \end{array} \right| =$$

$$= \frac{1}{3}x^2 \cdot e^{3x} - \frac{2}{3}\int e^{3x} x\,dx = \left| \begin{array}{ll} u = x & du = dx \\ dv = e^{3x}dx & v = \frac{1}{3}e^{3x} \end{array} \right| =$$

$$= \frac{1}{3}x^2 \cdot e^{3x} - \frac{2}{9}x \cdot e^{3x} + \frac{2}{9}\int e^{3x}dx =$$

$$= \frac{x^2 \cdot e^{3x}}{3} - \frac{2(x \cdot e^{3x})}{9} + \frac{2}{27}\int e^{3x}d3x =$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2 \cdot e^{3x}}{27} + C = \frac{e^{3x}}{3}\left(x^2 - \frac{2x}{3} + \frac{2}{9}\right) + C$$